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Do Banks Take Too Much Risk?

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Abstract

The paper evaluates the lending decisions of banks whose role is to sort out and finance profitable projects. Because established entrepreneurs can borrow directly from investors, banks tend to undervalue projects. Depositors cannot observe the lending policy of banks, which, on the other hand, gives banks incentive to exploit their limited liability and take too much risk. This moral hazard problem is reduced by future bank rents only to the extent that they constitute purely private bankruptcy costs. There is shown to be an inverse relation between private bankruptcy costs and social costs of bank failure, and that banks tend to take too little risk when private bankruptcy costs are high, and too much risk when these costs are low. Moreover, when private bankruptcy costs are high, new entry into banking may reduce risk-taking by banks, and a negative aggregate shock may result in a credit crunch.

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1 Introduction

Banks' risk-taking is the subject of considerable discussion. On the one hand, the banking sector is extensively regulated by most governments on the grounds that bank failures involve social costs and without regulation banks would take too much risk. On the other hand, one important activity of banks is to provide finance to small new firms, which due to information problems cannot finance themselves in the more anonymous security markets, and these firms often complain that banks are too conservative.[1] Especially in recessions, banks are criticized for refusing credit to profitable firms.[2]

Although banks' risk-taking has been the subject of much analysis, there are few attempts to analyse its implications for the credit allocation. In the literature analysing the efficiency of bank lending decisions, on the other hand, risk typically plays no role, as banks are treated as being perfectly safe, or as risk-neutral agents with unlimited liability.[3] Actual banks, however, are risky and have limited liability. Moreover, banks are mainly debt financed, suggesting that limited liability will influence their lending decisions.

First of all, the information processing role of banks implies that banks have private information about their borrowers, and hence the value of some bank assets cannot be fully recovered in a bankruptcy. This means that *bank owners will face bankruptcy costs*, which in turn suggests that banks will act as if they have risk-aversion. On the other hand, the private information of banks implies that banks' debt holders cannot observe the riskiness of the bank portfolios, and it is well-known from the corporate finance literature that this provides the owners of the banks with a taste of risk, the so called *moral hazard problem of debt financing* (Jensen and Meckling (1976)). The same type of risk-incentive has been shown to arise from the fixed-rate deposit insurance system applied by many countries (Merton (1977)).

There is some empirical evidence that banks in general do not exploit their limited liability,[4] suggesting that the "risk-aversion" arising from bankruptcy costs dominates the moral hazard problem (Bhattacharya (1982), Marcus (1984), and Keeley (1990)). Bankruptcy costs do not only reduce the bank's incentive to take excessive risks, but might also provide an explanation for banks refusing credits to projects with positive

[1] There is evidence that bank loans are not perfect substitutes for public debt. James (1987) reports that bank loans increase the value of the borrowing firms, and in Lummer and McConnel (1989), announcements of loan renewals increase the share prices of the borrowing firms. Slovin, Sushka, and Polonchek (1993) find that the impeding insolvency of the Continental Illinois Bank had negative effects, and the FDIC rescue positive effects, on the share prices of the bank's loan customers. The effects were strongest for firms with no publicly traded debt.

[2] For references see Rajan (1994).

[3] Bhattacharya and Thakor (1993) survey both literatures.

[4] Santomero and Vinso (1977) argue that the bankruptcy risk for a typical US bank seems to be almost negligible, and Marcus and Shaked (1984) find that most US banks pay an overprice for the deposit insurance.

net present value (NPV). A loan to a project may increase the probability that the bank will fail, although this project has a positive NPV. Hence, if the bank worries about going bankrupt it might not be willing to finance every project with positive NPV. This is especially relevant for new business that often have a negative expected return during the starting-period, which is compensated for by a high return in the case that the business turns out to be a success.

Another reason for banks to refuse credits to positive NPV projects is that *firms cannot commit to share future surplus with the bank.*[5] The information processing role of banks implies that banks will have long-term relationships with their loan customers, but for various reasons these relationships typically do not take the form of binding long-term contracts.[6] Hence, if the firm once it has succeeded has access to alternative sources of finance, the bank may not expect to get a sufficiently large share of the surplus in the future in order to be willing to lend to the firm when it is young and has a low expected return.[7]

On the other hand, if there are *social costs of bank failure*, the bank should not finance every project with positive NPV. If the return of the project in the beginning of its life-time is sufficiently low for the loan to contribute positively to the probability that the bank will fail, it might be better for the society if the bank does not fund this project, although it has a positive NPV. This implies that the bank actually may take too much risk although it is only financing projects with positive NPV. Moreover, whether the project should be funded or not depends on the financial position of the bank, and therefore the bank's credit policy should change over the business cycle. Hence, not only the lending decisions of banks, but also the benchmark against which they should be evaluated, is affected by the limited liability of banks.

The purpose of this paper is to evaluate the lending policy of banks with limited liability, whose role is to evaluate and monitor new firms and to allocate credit to profitable firms. The bank finances its lending with deposits from investors,[8] and a lending policy is more risky if it involves a higher probability of bank failure. There

[5]For discussion see Mayer (1988).

[6]One reason is regulation; banks' equity holdings are restricted in most countries, and many countries also restrict the maturity of bank loans.

[7]Analysing the data obtained from the National Survey of Small Business Finances Petersen and Rajan (1995) find that young firms in concentrated banking markets receive more institutional finance than do similar firms in more competitive markets. They also find that creditors seem to smooth interest rates over the life cycle of the firm in a concentrated market. They interpret this finding as banks in concentrated markets get a larger share of the firm's future return and therefore are more willing to lend to the firm when it is young.

[8]In this paper, I take the debt financing of banks as given. However, debt financing is known to reduce agency costs of delegation. For instance Diamond (1984) and Williamson (1986) show that the debt contract minimizes incentive costs when there is an incentive problem ex post to tell the truth about the portfolio outcome, and Cerasi and Daltung (1995) show that debt financing strengthens the incentive of the intermediary to evaluate projects, when project evaluation involves a private cost.

are two set of issues considered in the paper. First, do banks take too much or too little risk from a social point of view? Secondly, how does the lending policy of a bank depend on its environment, in particular on the number of competing banks, the competition from the security market, and the state of the economy?

The starting-point is the idea that new firms due to information problems must turn to banks for financing. Some information about the quality of the firm cannot be learned in advanced, but only experienced by letting the firm operate. Hence, in order to help all good firms to get on, banks will also have to finance some less good firms for a period of time. This means that, even if banks do offer loans with negative expected return to new customers, we cannot conclude that the bank is taking too much risk, because there is reason for subsidizing young firms.

On the other hand, when the firm has been operating for a while, more information about it has become public and as a result the firm is able to raise funds in the security markets. Hence, established entrepreneurs can borrow directly from investors. An established entrepreneur, however, may prefer to stay with his bank, as this bank has more information about him than do other lenders, and therefore may be able to offer loans on better terms. Hence, as in Sharpe (1990), customer relationships arise endogenously and banks are able to earn informational rents on their older customers. The possibility of banks to appropriate rents in the future, however, is limited by the competition from the security markets. As a result banks tend to undervalue projects.[9]

Both private and social costs of bank failure arise endogenously in the model, because banks get private information about their loan customers through the lending process, and this information is lost in a bank failure.[10] The private cost is given by the extent to which the information allows the bank to extract rents from the borrowers. The social cost, on the other hand, is given by the extent to which the information enables the bank to improve upon the credit allocation by resolving an adverse selection problem. The analysis shows that these costs do not need to coincide. For instance, it is possible that the information is privately valuable to the bank and the borrowers, even if it does not increase total surplus and therefore has no social value. On the other hand, the bank may not be able to extract the full value

[9]Also entrepreneurs have limited liability preventing banks from appropriating future surplus of the project in the beginning of the relation. Sharpe (1990) argues that incomplete contracts lead to over-investment as banks compete away future informational rents by lending to new borrowers at interest rates which initially generate expected losses. Here, however, the function of the bank is to sort out good projects, which implies that the bank should make an expected loss on the marginal loan in the first period.

[10]I assume that, if the bank cannot repay its debt, it is forced into bankruptcy, and the bankruptcy procedure hinders the bank from continuing in operation. Moreover, I assume that as in Chan, Greenbaum, and Thakor (1986) borrower information is non-transferable across banks so that the information is only reusable if it is available to a solvent bank. Gorton and Winton (1995) argue that when there are social costs of bank failure it may be optimal for the government to reduce capital constraints in order to keep the bank alive. I discuss this in the concluding section.

of improved credit allocation, but has to share the surplus with the borrowers. How much of the surplus the bank can extract depends on the structure of the financial markets.

The paper shows that independent of the market structure *the relation between private and social costs of bank failure is inverse* in the following sense: high private bankruptcy costs imply lower social costs than do low private bankruptcy costs. The intuition is the following: large private bankruptcy costs imply that the bank has low incentive to take risk, and since firms must get a bank loan to get started there will be only high quality firms around, which reduces (possibly eliminates) the adverse selection problem in the credit market.

On the other hand, *there is a positive relation between private bankruptcy costs and the undervaluation due to lack of commitment.* As said above, high private bankruptcy costs imply that the banks' customers will be of rather high quality. Since high-quality borrowers easier can get loans elsewhere, the undervaluation problem will be worse.

The role of the bank as delegated monitor implies that the depositors cannot observe the riskiness of the bank's lending activities, although they have rational expectations about the bank's behaviour.[11] This provides the bank with an incentive to take too much risk. *This risk-incentive is independent of the size of private bankruptcy costs.*

If the private bankruptcy costs are sufficiently large they will dominate the risk-incentive in the sense that the bank would like to minimize the variance of the portfolio return. However, even if bankruptcy costs do reduce bank owners' risk-incentive, they may not reduce the moral hazard problem of debt financing. *If the bank rents also represent social costs of bank failure, the moral hazard problem remains.*

In the model, banks can take either too much or too little risk. Because of the above discussed relations between private bankruptcy costs on the one hand and social costs of bank failure, undervaluation, and moral hazard problem on the other hand, the bank is more likely to take too little risk when private bankruptcy costs are high, and too much risk when private bankruptcy costs are low. For instance, high private bankruptcy costs imply larger undervaluation problem, but lower social costs of bank failure, so that for a given moral hazard problem there is a larger tendency towards underinvestment by banks.

Whether there is a so-called "credit crunch" in the banking sector is a recurrent subject of discussion in recessions.[12] This is not an easily settled issue, as banks should contract their lending, if the recession implies that firms' net present values

[11] I assume that deposits are uninsured. However, given that all outsiders the insurer inclusive have the same information about the bank's behaviour, the bank's risk-incentive would not be affected by the introduction of deposit insurance, if the insurance is fairly priced in expectation (see Daltung (1995)).

[12] For discussion, see Bernanke and Lown (1991) and Rajan (1994).

are reduced. Thus, the issue is whether lending is reduced more than is motivated by the reduced profitability of firms. To shed light on this issue, I analyse the impact of an aggregate shock affecting the profitability of all projects on the risk-incentive of banks.

It turns out that an aggregate shock has two counteracting effects on banks' risk-incentive. On the one hand, it aggravates the moral hazard problem of debt financing. The reason is that a negative shock reduces current rents in banking, which constitute an implicit bank capital. Hence, lower current rents imply that bank owners have less to lose on a risky strategy.[13] On the other hand, a negative shock increases the expected bankruptcy costs, as it increases the probability of bank failure, which reduces the risk-incentive of the bank. It is shown that for sufficiently high future rents, the last effect dominates and banks become more conservative after the economy has been hit by a negative shock. Hence, if banks due to high private bankruptcy costs take too little risk in normal times, there is a credit crunch in bad times in the sense that profitable projects that would have been granted credit in normal times now are rejected credit.

It is often claimed that increased competition in banking induces banks to take larger risks.[14] Mostly, this is considered as a problem, but increased competition has also been suggested as a solution to banks being too conservative in their lending. In order to shed some light on this issue, I evaluate the effect of increasing the number of banks. As a result of the information structure in the model, banks do not compete with loan rates for each other's customers, so the effect of increasing the number of banks is that each bank gets fewer customers. This is one aspect of increased competition.[15]

As in Diamond (1984), diversification is important to the incentive of the inter-

[13] As in Holmstrom and Tirole (1993) a reduction of bank capital has an adverse incentive effect. In contrast to this paper, however, they assume that investors through observing the amount of lending can infer the riskiness of the bank portfolio. Therefore, in their case the bank is forced to contract lending when the amount of bank capital is reduced.

[14] This view has some support in the literature. Chan, Greenbaum, and Thakor (1986) argue that increased competition reduces the return on screening which results in reduced asset screening and impaired bank asset quality. Keeley (1990) finds empirical support for banks with more market power taking less risk.

[15] In the model, the evaluation outcome is private information to the bank and the loan applicant, and the outcome is not verifiable. Furthermore, project evaluation is costly (both to the bank and the loan applicant), so that each entrepreneur is evaluated by only one bank. This results in a different type of competition than that analysed by Broecker (1990) and Riordan (1993) who also examine the evaluation function of banks. These authors assume that banks can commit themselves to interest rates before they evaluate projects, and that evaluation does not involve any costs, so that every entrepreneur is evaluated by all banks. They focus on imperfect evaluation outcomes, and show that due to the "winner's curse" of common value auctions new entry into banking might reduce the efficiency of bank lending. Chiesa (1994) shows that if each firm is evaluated by only a couple of banks that are capital constrained, ex ante price competition may lead to credit rationing, with the level of rationing increasing with the fragmentation of the banking system.

mediary. Here diversification reduces the moral hazard problem of debt financing, because the safer the bank is, the more likely it is that the bank has to bear the cost of a failing project itself. Hence, as new entry into banking implies that each bank becomes less diversified, it aggravates the moral hazard problem of debt financing. However, since diversification reduces the probability of bank failure, less diversification implies higher expected bankruptcy costs, which tends to reduce the risk-incentive of the bank. As in the case of a negative shock, if future rents are sufficiently large the latter effect may dominate and banks may become more conservative after new entry. Since it is when future rents are high that the bank is likely to take too little risk, entry seems to be a bad solution to this problem: the amount of lending may be reduced at the same time as the probability of bank failure is increased.[16]

The paper is organized in nine sections and two appendixes. The model is introduced in the next section. Financial intermediation is described in section 3. Financing of established entrepreneurs is analysed in section 4, and bank lending to new entrepreneurs in section 5. The question whether banks take too much risk is addressed in section 6. The effects of aggregate shocks on bank behaviour are analysed in section 7 and new entry in section 8. Finally, some concluding remarks are made in section 9. Proofs of propositions are relegated to appendix A. In appendix B, an alternative model specification of the competition in the second period is presented. The relation to the literature on bank-customer relationships is discussed during the course of the exposition.

2 The model

Consider a setting with three types of risk-neutral agents: entrepreneurs, investors, and banks. Each of m entrepreneurs has the opportunity to carry out an investment project with stochastic return. Projects take one period to carry out. Each project requires one unit of input in the beginning of the period. The project succeeds, and returns x units at the end of the period, with probability $(1-\delta)$. It fails, and returns nothing, with probability δ. Project outcomes are independent across entrepreneurs.

The values of x and δ differ among entrepreneurs. For each entrepreneur, x is drawn from a uniform distribution on $[0, X]$. Independent of x, δ takes one of two values, a low probability of failure, δ_l, with probability p_l, and a high probability of failure, δ_h, with probability p_h. Projects with a low probability of failure will be referred to as low-risk projects or just low-risks. Projects with a high probability of failure will be referred to as high-risks, and the value of δ as the type of project.

[16]The general view is that borrowers and savers are made better of by increased bank competition. Besanko and Thakor (1993), however, argue that, because increased competition leads to more risk-taking by banks, this may not be the case when customer relationships are valuable. This paper, on the other hand, show that when customer relationships are valuable, increased competition in form of new entry may neither increase risk-taking by banks nor improve borrowers' welfare.

There are likely to be many more high-risk projects than low-risk projects, that is $p_h > p_l$. Thus, the general case is that an entrepreneurial activity is a risky business, but there are some especially good ideas that are more likely to succeed.

If his project is a success, the entrepreneur has the option of continuing the project with the same values of x and δ for one more period. If the project is not carried out in period one, or if it fails, there is no possibility of carrying out the project in the second period. Entrepreneurs that have successfully carried out their projects in the first period are referred to as established entrepreneurs.

New entrepreneurs have no capital of their own, so they must borrow in order to carry out their projects. There is no shortage of capital; the total endowment of investors is much larger than m. Investors have access to an alternative investment technology that yields a certain return of y. They behave competitively, and are willing to lend their endowments, if they are offered a contract which gives them an expected return of at least y.

The distributions of x and δ are common knowledge, but initially nobody knows the value of x and δ for a given project.[17] Moreover, the expected average return of all projects in the economy is less than the alternative rate of return y. That is, if $\bar{\delta} = p_l\delta_l + p_h\delta_h$ is the expected average probability of failure among the population of new entrepreneurs, and $\bar{x}$ is the expected value of x, we have

Assumption 1 $(1 - \bar{\delta})\bar{x} < y$.

This means that project evaluation to sort unprofitable projects from profitable is necessary for new entrepreneurs to get project financing.

Project evaluation is costly both to the evaluator and to the entrepreneur. For simplicity, these costs are assumed to be non-pecuniary effort costs. If both the evaluator and the entrepreneur exert effort, they learn x, the return of the project in the success state. On the other hand, δ is learned through the investment process, so that at the end of the period both the entrepreneur and the financiers of the project know the value of δ. Neither the evaluation outcome, nor the type of the project, is freely observable or verifiable.

One interpretation of this information structure is the following: the x could be thought of as an objective characteristic of the project, the value of which should be possible to assert by drawing on a stock of knowledge of investment projects; the δ, on the other hand, could be thought of as capturing the ability of the entrepreneur to carry out his project, which can be learnt about only through trial.

Project outcomes are freely observable. Thus, at the beginning of the second period everybody knows the value of x for each established entrepreneur, which makes direct lending possible in this period.

[17]The results are not changed by the entrepreneur having private information about his project.

It is only possible to write binding contracts based on limited liability. If the required repayment is larger than the project outcome, the borrower goes bankrupt. It is not possible to write binding contracts for loans in the future.

3 Financial intermediaries

The role of banks is to evaluate and finance projects on the behalf of small investors (depositors), who neither have the incentive nor the knowledge to evaluate projects themselves.[18] The contract between the bank and the small investor is a one-period debt contract, and if the bank fails to repay the debt in the first period it goes bankrupt and is prohibited from operating in the second period.[19] For simplicity, banks have no capital of their own.

There is a cost of financial intermediation, which for simplicity is a constant cost per invested unit.[20] This cost manifests itself in that of D units of deposits only $\frac{D}{1+a}$ will be invested in projects, where $a > 0$. The cost of intermediation is small, especially

Assumption 2 $\left(1-\bar{\delta}\right)(1+a) \leq 1-\delta_l$.

This will imply that the bank is able to earn an informational rent in the second period.

There are n banks, where n is small compared to m, the number of entrepreneurs. Entrepreneurs apply to banks at the same time as banks determine their lending policies (those projects that shall receive credit). This means that the bank bases its decision about the lending policy on which entrepreneurs it expects shall apply and what loan rates it expects to receive.

Define two return functions of the bank, $R_1(x,\delta)$ and $R_2(x,\delta)$, where $R_t(x,\delta)$ is the loan rate in period t on a loan to a project with success outcome x, and default

[18] There could be some large investors that are willing to evaluate projects themselves, but who are not able to finance every profitable new project, so that m are the numbers of entrepreneurs that need bank financing.

[19] Since I want to focus on the effect of bankruptcy cost on bank behaviour, I avoid the issue whether investors have incentive to renew credit after renegotiation by assuming that investors wish to consume at the end of the period. Allowing for renegotiation would presumingly reduce bankruptcy costs, but not eliminate them.

[20] This cost could represent for instance the incentive cost of financial intermediation analysed by Diamond (1984), and Williamson (1986). In these papers, only the banker can freely observe the outcome of the bank's lending and must be provided with incentive to tell the truth to investors. With debt this incentive problem only arises in the bankruptcy states. However, in order to keep the analysis of the bank portfolio choice tractable, I will assume that the cost of intermediation occurs in every state, and not only in the default states. This simplifies the calculations, without affecting the results. The cost of financial intermediation will be of importance to the second period equilibrium.

probability δ. These functions capture how the entrepreneur and the bank share the surplus of the project, and they may be thought of as resulting from bargaining between the bank and the entrepreneur. Instead of stipulating a specific sharing rule, I allow for a general function. However, the optimizing behaviour of agents and the competition among lenders will impose some restrictions on these functions. I assume that the bank and the investors have correct beliefs about these functions, and I only consider the symmetric case in which these functions are the same for every bank.[21]

The gross portfolio return of the bank, denoted by z, is a random variable, and the distribution of z depends on the lending policy of the bank. Denote the expected portfolio return by $\bar{z}$, and the expected variance of the portfolio return by σ^2, where $\bar{z}$ and σ^2 are functions of the lending policy. I assume that all agents treat the normalized portfolio return, $\tilde{z}$, defined by,

$$\tilde{z}(z) = \frac{z - \bar{z}}{\sigma}, \tag{1}$$

where $\sigma = \sqrt{\sigma^2}$, as having a standard normal distribution.[22]

If the bank cannot meet its obligation to its depositors, that is if z is less than $r_d q$, where r_d is the promised return on bank debt (henceforth the deposit rate) and q is the investment volume, it goes bankrupt and the portfolio return belongs to its depositors. Hence, the probability that the bank goes bankrupt can be expressed as $\Phi(\tilde{z}(r_d q))$, where Φ is the cumulative distribution function of a standard normal random variable, and the expected return on deposits can be written as

$$r_d(1 - \Phi(\tilde{z}(r_d q))) + \frac{1}{q}\int_{-\infty}^{r_d q} z d\Phi. \tag{2}$$

The depositor will receive r_d only if the bank does not fail. If the bank fails, depositors share the bank portfolio return.

For simplicity, r_d is measured as promised return per invested unit. This implies that in order for investors to be willing to fund the bank the expected return in 2 must equal $y(1 + a)$. By integrating the second term in 2 by parts, the IR condition of depositors can be written as

[21] I do not analyse whether the sharing rule is socially optimal. In the model, the bargaining power affects the probability of bank failure, but it does not distort investments. Rajan (1992) show that bargaining power of the bank may have negative welfare effects in that it induces the entrepreneur to provide less effort.

[22] The variable z is a sum of independent random variables, the outcomes of the loans. According to the Central Limit Theorem for Independent Random Variables the distribution of $\tilde{z}$ approaches the standard normal distribution as the number of loans approaches infinity (see *e.g.* Ross (1976)). Since I assume that the number of loans are large, but finite, $\tilde{z}$ is only approximately normal. However, by increasing the number of loans and simultaneously reducing the profitability of the loans, the approximation can be improved at a non-negligible probability of bank failure.

$$r_d - \frac{1}{q}\int_{-\infty}^{r_d q} \Phi(\tilde{z}(z))dz = y(1+a). \tag{3}$$

The integral term in 3 represents the expected shortfalls of the debt.

In the same way the profit of the bank, which is equal to the excess returns in the non-default states,

$$\pi = \int_{r_d q}^{\infty} (z - r_d q) d\Phi,$$

can be rewritten as

$$\pi = \bar{z} - r_d q + \int_{-\infty}^{r_d q} \Phi(\tilde{z}(z))dz. \tag{4}$$

Depositors cannot observe, and the deposit rate cannot be contingent on, the lending policy of the bank. Depositors thus have to base their decisions, whether or not to make deposits in the bank at a given deposit rate, on their beliefs about the lending policy. Hence, r_d is independent of the actual lending policy of the bank. From 4 we see that, for a given expected portfolio return, and a given deposit rate, the profit of the bank increases with the spread in the portfolio return. This is the familiar point made by Stiglitz and Weiss (1981), and it is related to the point made by Merton (1977), that with a fixed price on deposits the bank has incentive to maximize the variance of the portfolio in order to maximize the expected return of bank owners. Thus, because the deposit rate cannot be contingent on the lending policy of the bank, the debt contract gives the bank a taste for risk.

On the other hand, a failure in the first period prevents the bank from operating in the second period, so future profits are lost if the bank goes bankrupt. Hence, the present value of the bank's profits in the first period is equal to

$$\Pi = \pi_1 + (1 - \Phi(\tilde{z}_1(r_{d1}q_1)))\,\pi_2, \tag{5}$$

where subindex t, $t\epsilon\{1, 2\}$, refer to period t. If the bank expects positive profits in the second period, the bank faces an expected cost of bankruptcy equal to $\Phi(\tilde{z}_1(r_{d1}q_1))\pi_2$ in the first period. This bankruptcy cost tends to make the bank behave as if it had risk-aversion. Hence, in the first period, there is a counteracting factor to the risk-incentive arising from the debt contract.

In order to choose the lending policy that maximizes 5 the bank has to form its belief about π_2. Let us therefore start with the second period.

4 Financing of established entrepreneurs

For established entrepreneurs, x is publicly known, which means that investors are willing to lend directly to these entrepreneurs. Banks and investors offer contracts to entrepreneurs simultaneously. Investors and outside banks face an adverse selection problem, since they do not know the type of the project, which is key information when determining the expected return on the loan. For all loan rates less than x, the probability that the entrepreneur defaults on his loan is equal to the probability that his project fails. Thus, for a given loan rate, a high-risk entrepreneur is more likely to default on his loan than a low-risk entrepreneur, so that the lender's expected return is higher on a low-risk loan than on a high-risk loan.

The analysis in this section is based on the assumption that the entrepreneur consumes all of his first period return at the end of the period, so that the entrepreneur has no capital of his own in period 2 either. This means that it is impossible for investors to sort borrowers into risk-classes. In appendix B, I analyse the case when the entrepreneur invests some of his first period return in durable goods, which he can pledge as collateral in the second period. As shown by Bester (1985) and Besanko and Thakor (1987), collateral requirements can be useful to sort borrowers into risk-classes. If sorting is costly, results do not depend on whether or not investors are able to sort established borrowers into risk-classes, but sorting implies higher social costs of bank failure.[23]

Investors compete for contracts and are willing to lend to established entrepreneurs given that they get an expected return of y. I will refer to the interest rate offered by investors as the capital market rate. Banks have a cost disadvantage vice vie investors, as intermediation is costly, but on the other hand banks have better information about its borrowers than do investors (and outside banks). Especially, the bank knows the type of each of its loan customers. According to assumption 2 the cost of financial intermediation is sufficiently low for there to be a positive informational rent. I will focus on the case when the bank is able to appropriate a share of this rent, so that the bank will earn positive profits on its low-risk customers.[24]

When it comes to customers of other banks, banks have no informational advantage vice vie investors. Prudently behaving banks therefore do not make competing offers to other banks' customers. However, as discussed in the previous section, since depositors cannot observe the lending policy of the bank, it is possible that the bank wants to trade-off a negative expected net return against an increased spread in the portfolio return.[25] I will show that this incentive is mitigated by diversification of

[23]This suggests that the results would also hold if the entrepreneurs could self-select by choice of investment size as in Innes (1991).

[24]More precisely, I assume that $\frac{y(1+a)}{1-\delta_l} < R_2(x, \delta_l) < \frac{y}{1-\delta_h}$, for $x > \frac{y(1+a)}{1-\delta_l}$. If there are any low-risks with $x < \frac{y(1+a)}{1-\delta_l}$, the bank would not finance them as depositors can observe x. Note that the second period interest rate is more constrained than the first period interest rate, which is only constrained to be less than x. This is in line with the finding of Berger and Udell (1995) that borrowers with longer banking relationships pay lower interest rates.

[25]One could argue that depositors should be able to observe whether a bank lends to customers of other banks, and the bank therefore would not do that. However, the bank may still want to make competing offers to its own high-risk borrowers, exploiting that depositors cannot observe the type

the bank portfolio, so that there is a size of the bank above which the bank is well-behaving in the sense that it is not competing for loans for which it has a comparative disadvantage.

Assume first that no bank has gone bankrupt in the first period. I will discuss the effect of bank failures afterwards. As will be shown, for some parameter values of the model there then is an equilibrium which is such that investors offer loans at the interest rate $\hat{r}_h$, defined by

$$\hat{r}_h = \frac{y}{1-\delta_h}, \tag{6}$$

to every established entrepreneur for which $x \geq \hat{r}_h$. Banks pursue a prudential lending policy which is such that the bank offers loans to its own low-risk borrowers at interest rates which are lower than $\hat{r}_h$, but makes no competing offers to its high-risk borrowers or to customers of other banks. Moreover, each bank sets a sufficiently high deposit rate for investors to be willing to fund the bank, given that they expect the bank to follow a prudential lending policy.

Investors have no incentive to deviate from the lending strategy above, when there are expected to be sufficiently many high-risk entrepreneurs around for the expected return on a loan contract which attracts low-risks to be negative.[26] Moreover, given that the bank is pursuing a prudential lending policy, investors have no incentive to refuse the bank credit. It remains to be shown that banks have no incentive to deviate from the prudential lending policy.

Proposition 1 *There is a size of the bank in terms of number of loans above which the bank does not have the incentive to deviate from the prudential lending policy given that depositors believe that the bank follows such a policy and the capital market rate is equal to* $\hat{r}_h$.

Proof: See appendix A.

Proposition 1 implies that the incentive of the bank to exploit its limited liability depends on how well diversified the bank is. The intuition is that a safer bank puts more weight on the expected return of the loan and less weight on the increase in the spread of the portfolio return than does a more risky bank, since it is more likely that a safer bank has to bear the cost of a failing project itself. Since the cost of financial intermediation implies that there is a discrete cost of exploiting limited liability and undercutting the offer by investors, there is a size of the bank above which the cost outweighs the benefit of such a deviation.

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of the borrower.

[26]For example, if $R_2(x, \delta_l) = R < \hat{r}_h$, for all x, this is true when $\hat{p}_h(1-\delta_h)R + \hat{p}_l(1-\delta_l)R < y$, where $\hat{p}_t$ is the updated probability that the project is of type t given that it has succeeded in period 1.

Remark 1 *As in Diamond (1984) diversification plays a role although all agents are risk-neutral. In Diamond diversification reduces private bankruptcy costs. Here it provides incentives for prudent lending behaviour ex ante.*

In the following, I will assume that banks are sufficiently large for it to be optimal for them to follow a prudential lending policy. That is, banks do not find it worthwhile to bid for customers about whom they have no informational advantage. This especially means that banks do not compete for each other's customers, and they do not finance any high-risk projects.

What happens if a bank goes bankrupt in the first period? Then investors will expect the entrepreneurs, who borrowed from this bank in the first period, to have both high-risk and low-risk projects, and they will offer them loans at a loan rate $\bar{r}$, which is larger than $y/(1-\delta_l)$ but smaller than $\hat{r}_h$.[27] Having the same information as investors, banks do not compete for these entrepreneurs.

To summarize, in the second period equilibrium low-risks borrow from the same bank as they borrowed from in the previous period, if the bank is still in operation. Hence,

Observation 1 *Low-risk entrepreneurs repeatedly borrow from the same bank, because that bank has better information about the entrepreneur, and therefore can offer loans on better terms.*

As in Sharpe (1990) bank-customer relationships arise endogenously as a result of asymmetric information. Such relationships, however, are established only between the bank and its low-risk customers. Because financial intermediation is costly, the bank must earn rents on its loans to be able to cover its costs.

Observation 2 *Banks lend only to low-risk customers, on which they can earn informational rents. Established high-risk entrepreneurs borrow directly from investors.*[28]

Finally, if a bank goes bankrupt, those customers that are able to pay the capital market rate borrow directly from investors, the others cannot carry out their projects in the second period. Since there are low-risk entrepreneurs with profitable projects,

[27] The results do not depend on how investors update their beliefs about the pool of entrepreneurs, or if they perform any updating at all, but a higher loan rate implies a higher potential social cost of bank failure.

[28] This seems at first to be contradictory to the results of Diamond (1991). In his paper firms build reputation by taking on costly bank-monitored debt, and when they have acquired sufficiently good reputation they borrow directly from investors. However, this is in fact in agreement with my financial structure, because Diamond assumes that high-risk projects are unprofitable projects, while low-risk projects are profitable projects. Thus, I take the analysis one step further in that I consider two types of profitable (and unprofitable) projects. I could add another period in which δ is known to investors and both high-risk and low-risk entrepreneurs raise funds in the capital market.

who cannot pay $\bar{r}$, and high-risk entrepreneurs with unprofitable projects who can pay $\bar{r}$, there is a potential social cost of bank failure.[29]

Let us now turn to the first period.

5 Bank lending to new projects

Each new entrepreneur applies for credit to a bank. As all banks have access to the same technology for project evaluation, the entrepreneur is indifferent concerning which bank to apply to, and the m entrepreneurs are expected to distribute themselves evenly among the n banks. Thus, all banks expect the same surplus from its lending business, and therefore choose the same lending policy.

After the entrepreneur has applied for credit, the bank evaluates the project.[30] The bank cannot commit itself to a loan contract before the evaluation, since the evaluation outcome is not verifiable. When the bank has evaluated the entrepreneur, the bank and the entrepreneur face a situation of bilateral monopoly, given that the project of the entrepreneur is sufficiently profitable for the bank to be interested in granting credit. The entrepreneur, who has learnt that he has a profitable project, cannot prove this to outside agents. The entrepreneur cannot even prove that he has been evaluated until he has signed a contract, which then is binding. A bank cannot observe whether a particular entrepreneur has applied to another bank, and an entrepreneur cannot observe what offer a bank gives to another entrepreneur.

Because every bank behaves in the same way in equilibrium, the entrepreneur applies for credit only at one bank. The only reason for applying for credit to yet another bank would be to get a better offer, as the second period equilibrium does not depend on the number of banks which have evaluated the project in the first period, and in equilibrium the entrepreneur expects to get the same offer from all banks. The outcome of the bilateral monopoly situation is given by $R_1(x, \bar{\delta})$, where $R_1(x, \bar{\delta}) \leq x$ because of limited liability of the entrepreneur.

Define marginal projects as the projects which the bank is only willing to finance if $R_1(x, \bar{\delta}) = x$, and the corresponding loans as marginal loans. Denote the success outcome of the marginal project by χ. Assume that loan rates on loans to projects with success outcomes higher than χ are higher than χ, that is $R_1(x, \bar{\delta}) > \chi, \forall x > \chi$. In this case, given that it is optimal to extend loans with expected return $(1 - \bar{\delta})\chi$, it is also optimal to grant credit to all projects with an expected return which is higher than the expected return of the marginal projects. The problem of the bank then is reduced to choosing the lowest value of x for which the bank is going to grant credit.

[29]There are empirical evidence that bank relationships are valuable. In particular, Petersen and Rajan (1994) find that for small business firms a relation with a bank increases the availability of credit.

[30]The evaluation costs are assumed to be sufficiently low for both the bank and the entrepreneur expecting evaluation to be profitable, given R_1 and R_2.

The lending policy determines the expected bank portfolio. As shown in appendix A, when the lending policy is to grant credit to all projects with success outcomes equal to, or larger than, χ, the expectation of the first period investment volume (the number of loans), the mean of the portfolio return, and the variance of the portfolio return are given by

$$q_1(\chi) = \frac{m}{n}(1-\frac{\chi}{X}), \tag{7}$$

$$\bar{z}_1(\chi) = \frac{m}{n}(1-\bar{\delta})\int_{\chi}^{X} R_1(x,\bar{\delta})\frac{1}{X}dx, \tag{8}$$

$$\sigma_1^2(\chi) = \frac{m}{n}\bar{\delta}(1-\bar{\delta})\int_{\chi}^{X} R_1(x,\bar{\delta})^2\frac{1}{X}dx + V(\chi), \tag{9}$$

and the normalized portfolio return in period one is given by

$$\tilde{z}_1(z;\chi) = \frac{z-\bar{z}_1(\chi)}{\sigma_1(\chi)}, \tag{10}$$

where $\sigma_1(\chi) = \sqrt{\sigma_1^2(\chi)}$. There are m entrepreneurs and n banks so that, disregarding indivisibilities, $\frac{m}{n}$ is the expected number of applicants per bank. Of these applicants, $\frac{m}{n}(1-\frac{\chi}{X})$ are expected to have projects with a larger return than χ, since x is uniformly distributed on $[0, X]$. The probability that a project has a return of x, is $\frac{1}{X}$, for all x. The expected return on such a loan is $(1-\bar{\delta})R_1(x,\bar{\delta})$, and the variance is $\bar{\delta}(1-\bar{\delta})R_1(x,\bar{\delta})^2$. The term $V(\chi)$ in 9, which is positive, arises because the m projects are drawn from a population of projects. Note that $\tilde{z}_1$ is also a function of χ.

Define

$$\rho(r_{d1},\chi) \equiv r_{d1}q_1(\chi) - \int_{-\infty}^{r_{d1}q_1} \Phi(\tilde{z}_1(z;\chi))dz - y(1+a)q_1(\chi). \tag{11}$$

Hence, $\rho(r_{d1},\chi)$ is the excess return on a deposit contract with deposit rate r_{d1} at the lending policy χ. From 4 follows that the first period profits can be written as

$$\pi_1(r_{d1},\chi) = \bar{z}_1(\chi) - \rho(r_{d1},\chi) - y(1+a)q_1(\chi). \tag{12}$$

The first period lending policy also affects expected second period profits, as it affects the stock of customers:[31]

$$\pi_2(\chi) = \frac{m}{n}p_l(1-\delta_l)\int_{\chi}^{X}[(1-\delta_l)R_2(x,\delta_l) - y(1+a)]\frac{1}{X}dx. \tag{13}$$

[31] I focus on the case where $\chi \geq \frac{y(1+a)}{1-\delta_l}$ in equilibrium. If $\chi < \frac{y(1+a)}{1-\delta_l}$, the bank does not finance the marginal project in the second period so in that case there is no effect on the customer stock of a marginal change in the lending policy.

With probability p_l the project is a low-risk project and succeeds in the first period with probability $(1-\delta_l)$. As the expectations of investors are fulfilled in equilibrium, the second period funding cost is equal to $y(1+a)$.

Given 7-13, the first order condition for profit maximization is

$$\begin{aligned} & - \frac{m}{n}\frac{1}{X}\left[\begin{array}{c} (1-\bar{\delta})\chi - y(1+a) \\ +(1-\Phi(\tilde{\omega}_1))p_l(1-\delta_l)\left((1-\delta_l)R_2(\chi,\delta_l) - y(1+a)\right) \end{array}\right] \\ & - \frac{\partial \rho}{\partial \chi} - \frac{d\tilde{\omega}_1}{d\chi}\varphi(\tilde{\omega}_1)\pi_2 = 0. \end{aligned} \qquad (14)$$

where $\tilde{\omega}_1 = \tilde{z}_1(r_{d1}q_{1;}\chi)$. By a marginal increase in χ, the bank expects to lose $\frac{m}{n}\frac{1}{X}$ customers. The term within the square brackets is the bank's expected net return on these customers. The expected net return on the marginal loan in the first period is $(1-\bar{\delta})\chi$ less the funding cost. The bank will finance the project in the second period *if* the bank repays it first period debt, the project turns out to be a low-risk project, and the project succeeds in the first period. The competition from the capital market in the second period restricts the second period loan rate and generally $R_2(\chi,\delta_l)$ will be less than χ. Since the deposit rate is not contingent on χ, a share of the benefit, $\frac{\partial \rho}{\partial \chi} > 0$, (or the loss, $\frac{\partial \rho}{\partial \chi} < 0$) from an increase in χ would accrue to depositors. Finally, the last term in 14 is the effect of marginal loans on the probability that the bank goes bankrupt. One can show that when marginal loans increase the variance of the portfolio return, this term is positive, that is marginal loans contribute positively to the probability of bank failure (see section A.3 in appendix A). This implies that

Observation 3 *The bank takes more risk in the sense of increasing the probability of bank failure by choosing a lower χ.*

There is no competition for deposits, so the bank sets the deposit rate as low as possible. Depositors cannot observe the lending policy of the bank so whether or not investors are willing to deposit their endowments at the bank at a given deposit rate depends on their *beliefs* about the bank's lending policy. The deposit rate must be such that, given the expectation of the investor, the return on deposits per invested unit is equal to $y(1+a)$. In a rational expectation equilibrium, the belief of investors is fulfilled. Hence, the first period rational expectation equilibrium is a pair ($\hat{\chi}$, $\hat{r}_{d1}$) that solves the equation system consisting of equation 14 and the IR condition of investors:

$$\rho(r_{d1},\chi) = 0.$$

I will focus on the case where $\frac{\partial \rho}{\partial \chi} > 0$ in equilibrium, that is the marginal loans increase the spread in the average portfolio return, as otherwise there is no reason for depositors to worry about banks taking too much risk.[32] Thus, the bank has lower incentive to increase χ (larger incentive to take risk) than it would have if the deposit rate was contingent on the lending policy of the bank. In fact, if $\frac{\partial \rho}{\partial \chi}$ is large the incentive of the bank to exploit the fact that depositors cannot observe its lending policy might be too strong for investors to be willing to finance the bank. In appendix A, I show that there exists an equilibrium provided that the bank is sufficiently large. The motivation is the same as for Proposition 1: the interests of a larger and more diversified bank are more aligned with the interests of depositors, as a large bank expects to carry a large part of the losses from bad loans itself. Or in other words, $\frac{\partial \rho}{\partial \chi}$ decreases with the size of the bank. In the following, I will assume that $\frac{m}{n}$ is sufficiently large for each bank to be sufficiently diversified for there to be an equilibrium.

In the short run the bank has incentive to take risk in order to exploit its limited liability: if the project is a success the bank gets the benefits, but if it fails the bank may also fail, and then the losses are borne by the bank's depositors. However, if the bank goes bankrupt, future profits are lost, which reduces the risk-incentive of the bank, and we have the following result:

Lemma 1 *There is a size of expected future bank profits above which a variable portfolio outcome induces the bank to invest less than it would if the outcome was certain.*

Proof: See appendix A.
If expected future profits are sufficiently large, the bank will become "risk-averse" in the sense that an increase in the spread of the bank portfolio *reduces* the present value of the expected bank profits, although it increases first period profits. This is known from Marcus (1984). The issue is, however, whether it is good from a social point of view that the bank is "risk-averse" in the sense of lemma 1. If the opposite of lemma 1 applies, that is, if the bank invests more than it would if the outcome was certain, I will say that the bank has a taste for risk, and the question is if this is bad?

6 Optimal risk-taking by banks

In this section I analyse whether banks given the financial structure take too much or too little risk from a social point of view. The expected social surplus created by the bank is given by

$$W(\chi) \;=\; S_1(\chi) - y(1+a)q_1(\chi) \tag{15}$$

[32]Moreover, it will be shown that, if $\frac{\partial \rho}{\partial \chi} < 0$, the model banks never take too much risk from a social point of view, which is what worries bank regulators.

$$+\left[\begin{array}{c} S_{l2}(\chi) - y(1+a)q_{l2}(\chi) \\ +S_{h2}(\max(\hat{r}_h, \chi)) - yq_{h2}(\max(\hat{r}_h, \chi)) \end{array}\right](1-\Phi(\tilde{\omega}_1))$$
$$+\left[\begin{array}{c} S_{l2}(\max(\bar{r}, \chi)) - yq_{l2}(\max(\bar{r}, \chi)) \\ +S_{h2}(\max(\bar{r}, \chi)) - yq_{h2}(\max(\bar{r}, \chi)) \end{array}\right]\Phi(\tilde{\omega}_1),$$

where

$$\tilde{\omega}_1 = \tilde{z}_1(\tilde{r}_{d1}(\chi)q_1(\chi); \chi); \quad (16)$$
$$S_1(a) = \frac{m}{n}(1-\bar{\delta})\int_a^X x\frac{1}{X}dx; \quad (17)$$
$$q_{t2}(a) = \frac{m}{n}p_t(1-\delta_t)(1-\frac{a}{X}),\ t\epsilon\{h,\ l\}; \quad (18)$$
$$S_{t2}(a) = \frac{m}{n}p_t(1-\delta_t)^2\int_a^X x\frac{1}{X}dx,\ t\epsilon\{h,\ l\}; \quad (19)$$

and $q_1(\chi)$ is defined by 7. $\tilde{r}_{d1}(\chi)$ is the first period deposit rate required to give depositors an expected return equal to y, when the lending policy is given by χ. $S_1(a)$ is the expected first period surplus of all projects with success return equal to or larger than a. With probability p_t an entrepreneur of type t succeeds in the first period and survives to the second period. Thus, $q_{t2}(a)$ is the expected number of projects of type t with a success outcome larger than a in the second period, and $S_{t2}(a)$ is the second period expected surplus of these projects.

The terms in 15 can be explained as follows. In the first period, projects are financed with bank loans at the social funding cost $y(1+a)$. If the bank is still in operation in the second period, low-risk projects, are financed with bank loans, while high-risk projects with x larger than $\hat{r}_h$ are financed with direct loans from investors at the social funding cost y. If the bank goes bankrupt, those of its customers that can pay $\bar{r}$ borrow directly from investors in the second period at the social funding cost y.

The first question is whether the bank takes too much risk from a social point of view, when it has a taste for risk? The answer is given by the value of the derivative of W with respect to χ in equilibrium. The bank takes *too much risk* if it finances projects with a lower success outcome than is socially optimal. Accordingly, the bank takes *too little risk*, if projects that are socially optimal, taking into account that they may increase the probability of bank failure, do not get financing.

Assume that $\left(1-\bar{\delta}\right)(1+2a) > 1-\delta_l$. This implies that a perfectly safe bank would not choose $\hat{\chi} < \bar{r}$. Hence, if the bank chooses $\hat{\chi} < \bar{r}$, it means that the bank has a taste for risk. Does the taste for risk imply that the bank takes too much risk? Differentiating W with respect to χ under this assumption, and then substituting the equilibrium condition 14 into the first order condition for a socially optimal lending policy gives

$$
\begin{aligned}
\left.\frac{dW}{d\chi}\right|_{\hat{\chi}<\bar{r}} &= -\frac{m}{n}\frac{1}{x_M}\left[p_l(1-\delta_l)(1-\delta_l)\left(\hat{\chi}-R_2(\hat{\chi},\delta_l)\right)\left(1-\Phi(\tilde{\omega}_1)\right)\right] \\
&+\left[\begin{array}{c}\pi_2-\left(S_{l2}(\hat{\chi})-y(1+a)q_{l2}(\hat{\chi})\right)+\left(S_{l2}(\bar{r})-yq_{l2}(\bar{r})\right) \\ +\left(S_{h2}(\bar{r})-yq_{h2}(\bar{r})\right)-\left(S_{h2}(\hat{r}_h)-yq_{h2}(\hat{r}_h)\right)\end{array}\right]\varphi(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{d\chi} \\
&+\frac{\partial\rho}{\partial\chi}-\pi_2\varphi(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{dr_{d1}}\frac{dr_{d1}}{d\chi}.
\end{aligned}
$$

The term within the first square brackets is positive[33]: the bank tends to take too little risk because it does not consider the entire second period surplus of the marginal projects as it has to share it with the entrepreneurs. For the same reason the expected future profits are lower than the expected second period surplus of the low-risk projects: the sum of the first two terms within the second square brackets is negative. On the other hand, most low-risk projects will be able to pay the capital market rate, creating a social surplus even if the bank fails. However, there are also high-risk projects with negative social surplus that are able to borrow in the market if the bank fails: the sum of the two last terms within the second square brackets is negative. The sign of the second square brackets depends on the parameters of the model. Finally, the last two terms, which arise because the bank does not take into account the effect on the deposit rate, tend to make the bank take too much risk.

We cannot generally say that the bank takes too much risk when it has a taste for risk. It will depend on how much of the second period surplus of the project the bank can appropriate and the difference between private and social bankruptcy costs. Note, that not even if the bank expects to earn zero profits in the second period, so that it does not face any costs of bankruptcy, it will necessarily take too much risk, as zero profits imply that the bank does not take any of the second period surplus of the project into account. In the model of appendix B, the bank appropriates the whole second period surplus of those entrepreneurs who have no alternative funding source in the second period. Moreover, the social cost of bank failure is equal to the private cost of bank failure. In that special case, the bank unambiguously takes too much risk when it has a taste for risk.

For similar reasons we cannot generally say that the bank will take too little risk when it is risk-averse (see appendix B), although it will be the case in this model, if $(1-\delta_h)(1+a) > 1-\bar{\delta}$:

Proposition 2 *If expected future profits are large, it is possible that the bank takes too little risk.*

Proof: Given that $(1-\delta_h)(1+a) > 1-\bar{\delta}$ it follows from the proof of lemma 1 that there is a size of future profits for which $\chi > \hat{r}_h$. For $\chi > \hat{r}_h$ the derivative of W with respect to χ in equilibrium is given by

[33] I consider the case when $\frac{y(1+a)}{1-\delta_l} < \hat{\chi} < \frac{y}{1-\bar{\delta}}$.

$$\frac{dW}{d\chi}_{\chi>\hat{r}_h} = -\frac{m}{n}\frac{1}{X}\left[\begin{array}{c} p_l(1-\delta_l)(1-\delta_l)\left(\hat{\chi}-R_2(\hat{\chi},\delta_l)\right)(1-\Phi(\tilde{\omega}_1)) \\ +p_l(1-\delta_l)((1-\delta_l)\hat{\chi}-y)\Phi(\tilde{\omega}_1) \\ +p_h(1-\delta_h)((1-\delta_h)\hat{\chi}-y) \end{array}\right] \quad (20)$$
$$+\frac{\partial\rho}{\partial\chi}-\pi_2\varphi(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{dr_{d1}}\frac{dr_{d1}}{d\chi}+(\pi_2+yaq_{l2}(\hat{\chi}))\varphi(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{d\chi}.$$

From the proof of lemma 1 follows that there is a size of future profits for which

$$\frac{\partial\rho}{\partial\chi}-\pi_2\varphi(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{dr_{d1}}\frac{dr_{d1}}{d\chi}+\pi_2\varphi(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{d\chi}<0,$$

in which case 20 is negative. *Q.E.D.*[34]

The reason is that social costs of bank failure are endogenous. When $\hat{\chi} > \hat{r}_h$, there are no social costs of bank failure in this model, because all low-risk entrepreneurs are able to borrow directly from investors in the second period, and there are no bad projects around. On the contrary, there is a positive effect on social welfare of the bank going bankrupt, namely that low-risks are now funded at a lower social cost. Thus, while the bank faces private costs of bankruptcy, the welfare cost is negative. Negative social bankruptcy costs are not necessary, however, for the bank to take too little risk. Zero social costs are more than sufficient, as the bank, unable as it is to appropriate all of the second period surplus, tends to undervalue marginal projects.

Remark 2 *In the model of appendix B, there are always positive social costs of bank failure, as low-risk entrepreneurs have to pay a sorting cost, but social costs are the lower the higher are the private bankruptcy costs. Hence, while the negative welfare cost must be considered as being model specific, the inverse relation between private and social bankruptcy costs is more general. High private bankruptcy costs imply that the marginal entrepreneur has a high-quality project, as the bank is "risk-averse", and therefore has access to alternative funding if his bank fails.*

To conclude, the bank in this paper sorts out and finances new projects. A bank loan is the only source of finance for a new entrepreneur, so if he does not get a bank loan, he cannot carry out his project. Therefore, the lending policy, and the degree of risk-taking by the bank, is important to social welfare. Whether the bank is taking too much or too little risk from a social point of view is determined by the balance of three factors: firstly, too which extent private bankruptcy costs represent social costs of bank failure; secondly, to which extent the bank can appropriate the second period surplus of the firm; and finally the severeness of the moral hazard problem. We cannot generally tell which factor dominates, since this depends on the exact model specification as well as on the parameters of the model. However, there is a stronger tendency towards too little risk-taking when future profits are large, and towards too much risk-taking when these rents are low.

[34] Note that if $\frac{\partial\rho}{\partial\chi} < 0$, independent of the size of π_2, the bank will take too little risk.

7 Aggregate shocks

How does banks' risk-taking depend on the state of the economy? Are there more important reasons to worry about banks' risk-taking in downturns than in upturns, or are there reasons to believe that there will be a credit crunch in downturns? In this section, I will analyse the effect on bank behaviour of aggregate shocks.

Let the expected return of the project in period 1 be equal to $(1-\delta)\theta x$, where θ is the same for all entrepreneurs and represents an aggregate shock. The economy can be in one of three states, $\theta \in \{\theta_b, 1, \theta_g\}$, which will be referred to as bad, normal, and good times. Hence, the previous analysis could be thought of as the lending policy in normal times. We can either think of θ as affecting the probability of success, the success return, or both. For simplicity, times are believed to be normal in period 2 with probability 1.

Assume that the economy is hit by a negative shock in period one, $\theta = \theta_b$. How does this shock affect the lending policy of the bank? Since projects now are less valuable, fewer projects should get financing, but what happens to the risk-incentive of the bank? It is perhaps by now no surprise that this will depend on the size of expected future profits:

Proposition 3 *There is a size of expected future profits above which the bank has less incentive to take risk in bad times.*

Proof: see appendix A.
There are two counteracting effects on banks' risk-taking of a negative shock. On the one hand, as the bank becomes less profitable, it has stronger incentive to exploit its limited liability. On the other hand, lower profits implies that the contribution of the marginal project to the probability of bank failure goes up, which tends to make the bank more conservative. If future expected profits are high, the last effect will dominate. Otherwise, the bank will have a stronger incentive to take risk in bad times.

Similarly, if the economy is hit by a positive shock, $\theta = \theta_g$, the bank becomes more profitable and have less incentive to exploit depositors, but expected bankruptcy costs go down and we have

Corollary 4 *There is a size of expected future profits above which the bank has more incentive to take risk in good times.*

To conclude, there is a size of expected future profits for which the bank is "risk-averse", and moreover the bank has less incentive to take risk in bad times than in good times. For sufficiently low future profits, on the other hand, the bank has a taste for risk, which is stronger in bad times. This, however, does not necessarily mean that low future profits imply that there are stronger reasons to worry about banks' risk-taking in bad times. The reason is that the bank tends to undervalue marginal projects, because it cannot appropriate the whole second period surplus, and this undervaluation problem is accentuated in bad times: the incentive of the bank to bridge over bad times is limited by the share of the bank in normal times.

Proposition 3 and Corollary 4 imply that, if banks due to high expected future profits take too little risk in normal times, there will be a credit crunch in bad times:

Proposition 5 *If expected future profits are large, there may be an under-investment problem, which is aggravated in bad times and reduced in good times.*

Proof : It follows from the proof of proposition 3, that there may be a size of expected future profits for which χ will be higher in bad times. Then, given that $(1-\delta_h)(1+a) > 1-\bar{\delta}$, there is a size of π_2 for which $\chi > \hat{r}_h$ according to lemma 1, and the derivative of the welfare function is given by

$$- \quad \frac{m}{n}\frac{1}{X}\left[\begin{array}{c} (1-\bar{\delta})\theta\hat{\chi} - y(1+a) \\ +(1-\Phi(\tilde{\omega}_1))p_l(1-\delta_l)\left((1-\delta_l)\hat{\chi} - y(1+a)\right) \\ +p_l(1-\delta_l)((1-\delta_l)\hat{\chi} - y)\Phi(\tilde{\omega}_1) \\ +p_h(1-\delta_h)((1-\delta_h)\hat{\chi} - y) \end{array}\right]$$
$$+yaq_{l2}(\hat{\chi}))\varphi(\tilde{\omega}_1)\frac{d\tilde{z}_1}{d\chi}. \tag{21}$$

According to the proof of proposition 3, there is a size of π_2 for which the sum of the first two terms are higher with $\theta = \theta_b$ than with $\theta = 1$, and the rest will also be larger as both χ and the probability of bank failure are larger in bad times. Similarly, if $\chi > \hat{r}_h$, 21 is smaller for $\theta = \theta_g$. *Q.E.D.*

Hence, if future expected bank profits are high, there may be a credit crunch problem in the sense that more socially valuable projects are rejected credit in bad times than in good times.

8 New entry into financial intermediation

It is often claimed that increased competition would induce banks to take larger risks, that is banks would finance projects that previously were rejected credit. This is mostly perceived as a problem, but increased competition has also been advocated by those who believe that banks are too conservative. In order to shed light on the effect of increased competition on banks' risk-taking, in this section I analyse how an increased number of banks affects the equilibrium lending policy of each bank.

An increased number of banks implies that each bank becomes smaller and riskier. This has two counteracting effects on banks' risk-taking, similar to the effects of a negative aggregate shock. On the one hand, the bank has a stronger incentive to exploit its limited liability, on the other hand, the contribution of the marginal project to the probability of bank failure is increased. However, in addition entry reduces the size of future profits, as it reduces the customer stock of each bank. Hence, expected bankruptcy costs may either increase or decrease as a result of new entry, but we have[35]

Proposition 6 *There is a size of expected future bank profits above which an increase in the number of banks implies that sufficiently large and profitable banks require a higher return on marginal loans in equilibrium.*

Proof: See appendix A.
Hence, if the bankruptcy cost of the bank is sufficiently large, new entry into banking leads to an increase in the average quality of bank loans.

As banks are likely to be too conservative, exactly when future profits are high, Proposition 6 implies that increased competition might be a bad solution to banks taking too little risk: the amount of lending may decline at the same time as the probability of bank failure is increased.

For lower expected future profits, entry induces the bank to reduce χ for a given deposit rate. The equilibrium deposit rate is increased, which counteracts this incentive, but for sufficiently low future profits the bank will take more risk. This is exactly in the case when banks are likely to take too much risk. Hence we have

Observation 4 *Entry tends to reduce risk-taking by banks when banks are taking too little risk from a social point of view. Similarly, entry is likely to increase risk-taking by banks in the case when they already take too much risk.*

9 Concluding remarks

There is considerable debate about banks' risk-taking. On the one hand, small firms often complain that banks are too unwilling to take risk. The banking literature, on the other hand, has focused on the moral hazard problem of debt financing, and regulators seem to believe that without regulation banks would take too much risk. However, to motivate regulation it is not sufficient to establish that banks have a taste for risk, as one role of the bank is to evaluate new projects, and to allocate credit to profitable projects, which inevitable involves risk-taking by the bank. In this paper I take the debt financing of the bank as given and analyse whether banks take too much or too little risk, explicitly taking into account the function of the bank as evaluator of projects.

[35] I assume that the return functions R_1 and R_2 are not affected by the entry. Given the information structure there is no reason for why the negotiation power of the bank would be affected by the new entry. If there were down-ward shifts in the return functions, all three effects of entry would be amplified.

In the paper there are three factors distorting the lending decisions of banks. Firstly, depositors cannot observe the riskiness of the bank's lending activities providing bank owners with a taste for risk. Secondly, it is not possible to write a binding contract which gives the bank a right to the second period return of the entrepreneur. Since established entrepreneurs will have an alternative source of finance, namely direct loans from investors, the bank will disregard some of the future surplus that is created by the bank's lending today, which tends to make the bank take too little risk. Finally, private bankruptcy costs may not coincide with social costs of bank failure. If the former is larger than the latter there will be a tendency towards too little risk-taking, and the other way around, if the latter is larger than the former there will be a tendency towards too much risk-taking.

It is well-known that bankruptcy costs, for instance in form of the loss of future profits, reduce the bank's incentive to take risk and might even make the bank "risk-averse" in the sense that an increased spread in the portfolio return reduces expected bank profits. This, however, does not, as often assumed, necessarily imply that the moral hazard problem of debt financing is reduced: if these bankruptcy costs also represent social costs of bank failure, the moral hazard problem remains. In this model both private and social costs of bank failure arise endogenously, and there is shown to be an inverse relation between them in the sense that high private bankruptcy costs imply lower social costs of bank failure than do low private costs. Moreover, the moral hazard problem is independent of the size of these cost, while the undervaluation problem due to lack of binding contract is positively related to the private bankruptcy costs. Together this means that there is a stronger tendency towards too little risk-taking when private bankruptcy costs are high and towards too much risk-taking when private bankruptcy costs are low.

For the same reason, there is more likely to be a credit crunch in recessions, if private bankruptcy costs are high. That is, if the bank worries about going bankrupt, it will be less inclined to bridge over bad times, and there will be more profitable projects that are refused credit in bad times than in good times. On the other hand, the moral hazard problem is aggravated in bad times, so if private bankruptcy costs are low there may be more reasons to worry about banks' risk-taking in bad times.

New entry into banking is often suggested as a solution to banks being too conservative from a social point of view. However, for similar reasons as for there to be a credit crunch in bad times, this might not be a good solution. If the private cost of bank failure is sufficiently large, banks worry about going bankrupt and, as a result of the increased probability of bank failure, will reject credit to entrepreneurs who before the entry would have received credit. Hence, in this case, the probability of bank failure increases, and banks become even more conservative.

Without doubt, the paper neglects several potentially important features of banking. I would like to stress, however, that most of the results do not hinge on the specific structure assumed here. For instance, the first order condition for profit maximum can be evaluated for portfolios consisting of more assets than loans to new or established entrepreneurs. I also believe that it is possible to introduce systematic risk, without undermining the framework. For instance, assume that the loan return consists of an idiosyncratic part and a systematic part. Furthermore, assume that the distribution of the systematic shock is normal and independent of the distribution of the idiosyncratic shock. Then, the portfolio return of the bank would still be normally distributed, but the variance of the portfolio return would not go to zero as the bank size increases, but it would approach the variance of the systematic shock. As long as this variance is not too large, the results of the paper would still hold true.

The paper does not consider bank regulation. It is, however, the same mechanism behind the risk-incentive arising from depositors' lack of information as the risk-incentives arising from a fixed-rate deposit insurance system, which since Merton (1977) has been the main focus of most of the literature analysing banks' portfolio choice and the impact of bank regulation on bank behaviour: a non-risked based deposit insurance gives rise to the same kind of risk-incentive as does a risk-insensitive deposit rate. However, the lending policy of the bank will depend on whether the deposit insurance is fairly priced.

The paper has taken the debt financing of the bank as given. As has been shown, it is the structure of the debt contract that gives the bank incentive to take too much risk. This does not necessarily motivate capital constraints, since in a competitive capital market with rational investors the bank bears the costs of excessive risk-taking itself. However, a fixed deposit insurance premium, which does not depend on the amount of bank capital, may motivate capital constraints as it provides the bank with incentive to increase leverage (Daltung (1995)).

Note that, if the bank is "risk-averse", capital constraints may actually increase risk-taking by the bank, even if they reduce the moral hazard problem, as capital reduces the expected bankruptcy cost of the bank. However, this may be socially beneficial, if high private bankruptcy costs imply that the bank takes too little risk from a social point of view.

Finally, it is often claimed that if the government is going to bail out failing banks in order to reduce social costs of bank failure, they should avoid bailing out the bank's shareholders, as they otherwise have too strong incentive to take risk. However, this policy is only desirable if shareholders already have too strong incentive to take risk. If they instead are taking too little risk, reducing social but not private costs of bank failure would aggravate the under-investment problem.

A Appendix

A.1 Proof of proposition 1

Partial integration gives

$$\int_{-\infty}^{r_d q} \Phi(\tilde{z}(z))dz = r_d q \Phi(\tilde{\omega}) - \int_{-\infty}^{r_d q} \frac{z}{\sigma} \varphi(\tilde{z}(z))dz, \tag{22}$$

where $\varphi(\cdot)$ is the density function of a standard normal variable and $\tilde{\omega} = \tilde{z}(r_d q)$. We have that

$$\int_{-\infty}^{r_d q} \frac{z}{\sigma} \varphi(\tilde{z}(z))dz = \int_{-\infty}^{r_d q} \frac{z - \bar{z}}{\sigma} \varphi(\tilde{z}(z))dz + \int_{-\infty}^{r_d q} \frac{\bar{z}}{\sigma} \varphi(\tilde{z}(z))dz. \tag{23}$$

Using that $\frac{d\varphi(x)}{dx} = -x\varphi(x)$, we can solve for the integrals on the right hand side of 23 and get

$$\int_{-\infty}^{r_d q} \frac{z}{\sigma} \varphi(\tilde{z}(z))dz = -\sigma\varphi(\tilde{\omega}) + \bar{z}\Phi(\tilde{\omega}). \tag{24}$$

Substituting 24 into 22 and the result into the profit function of the bank and the IR condition of depositors given by 4 and 3 respectively gives

$$\pi = (\bar{z} - r_d q)(1 - \Phi(\tilde{\omega})) + \sigma\varphi(\tilde{\omega}), \tag{25}$$

$$r_d(1 - \Phi(\tilde{\omega})) + \frac{\bar{z}}{q}\Phi(\tilde{\omega}) - \frac{\sigma}{q}\varphi(\tilde{\omega}) = y(1 + a). \tag{26}$$

The change in profits by the addition of a loan with expected return $\bar{w}_i$ and variance σ_i^2 is given by

$$d\pi = (\bar{w}_i - r_d)(1 - \Phi(\tilde{\omega})) + \sigma_i^2 \frac{1}{2\sigma} \varphi(\tilde{\omega}). \tag{27}$$

Substituting the IR condition 26 into 27 gives

$$d\pi = \bar{w}_i - y(1 + a) + (\frac{\bar{z}}{q} - \bar{w}_i)\Phi(\tilde{\omega}) + (\frac{\sigma_i^2}{2} - \frac{\sigma^2}{q})\frac{1}{\sigma}\varphi(\tilde{z}(z)). \tag{28}$$

To attract other borrowers than the bank's own low-risk customers, the bank must offer a contract which gives the bank an expected return that is no higher than y. The bank is willing to do that when the sum of the two last terms in 28 is larger than ya. Since $\frac{\bar{z}}{q} > y(1 + a)$ under the prudential lending policy, the first term of the sum is positive. Whether the second term is positive depends on the variance of the loan. In any case, under the prudential lending policy, the sum approaches zero as m approaches infinity; since $\frac{\bar{z}}{q} > y(1 + a)$, the equilibrium deposit rate approaches $y(1 + a)$ and $\tilde{\omega}$ approaches $-\infty$ as m approaches infinity. Hence, there is a size of the bank in terms of number of loans for which the bad loan contributes negatively to profits. Given that loans to other borrowers than the bank's own low-risk customers contribute negatively to bank profit, the bank has no incentive to deviate from the prudential lending policy. *Q.E.D.*

A.2 Derivation of the period 1 portfolio distribution

We have that $\frac{m}{n}$ projects, where $\frac{m}{n}$ is assumed to be an integer, are drawn from the population of projects. Let q be the number of projects with a success outcome equal to or larger than χ. Then, q is a binomial random variable with parameters $\frac{m}{n}$ and $1-\frac{\chi}{X}$, in short $q = Bin(\frac{m}{n}, 1-\frac{\chi}{X})$. Let X_i be the success outcome of project i, and Y_i be the return on a loan to project i in period 1. X_i is uniformly distributed on $[0, X]$ for $i = 1, 2, ..., \frac{m}{n}$. Let x_i be the realization of X_i. Given that $X_i = x_i$, $Y_i = R_1(x_i, \bar{\delta})Bin(1, 1-\bar{\delta})$, where $R_1(x_i, \bar{\delta}) = 0$ for $x_i < \chi$. We have that

$$E(Y_i|X_i = x_i) = (1-\bar{\delta})R_1(x_i, \bar{\delta}),$$

$$Var(Y_i|X_i = x_i) = \bar{\delta}(1-\bar{\delta})R_1(x_i, \bar{\delta})^2,$$

where E is the expectation operator and Var is the variance operator.

Let $Z_i = Y_i 1\{X_i \geq \chi\}$, where $1\{X_i \geq \chi\} = 1$, if $X_i \geq \chi$, and $1\{X_i \geq \chi\} = 0$, if $X_i < \chi$. Using that

$$E(Z_i) = E(E(Z_i|X_i)),$$

$$Var(Z_i) = Var(E(Z_i|X_i)) + E(Var(Z_i|X_i)),$$

gives

$$E(Z_i) = (1-\bar{\delta})\int_{\chi}^{X} R_1(x, \bar{\delta})\frac{1}{X}dx,$$

$$Var(Z_i) = \bar{\delta}(1-\bar{\delta})\int_{\chi}^{X} R_1(x, \bar{\delta})^2\frac{1}{X}dx + (1-\bar{\delta})^2\int_{\chi}^{X} R_1(x, \bar{\delta})^2\frac{1}{X}dx - E(Z_i)^2.$$

The portfolio outcome z is given by, $z = \sum_{i=1}^{\frac{m}{n}} Z_i$, and we have that $E(z) = \frac{m}{n}E(Z_i)$ and $Var(z) = \frac{m}{n}Var(Z_i)$, since Z_i are independent, which gives 7-9 where $V(\chi)$ in 9 is given by

$$V(\chi) = \frac{m}{n}[(1-\bar{\delta})^2\int_{\chi}^{X} R_1(x, \bar{\delta})^2\frac{1}{X}dx - E(Z_i)^2].$$

A.3 The existence of first period equilibrium

As follows from section A.1, the present value of profits, given by 5 can be written as

$$\Pi = (\bar{z}_1 - r_{d1}q_1)(1-\Phi(\tilde{\omega}_1)) + \sigma_1\varphi(\tilde{\omega}_1) + (1-\Phi(\tilde{\omega}_1))\pi_2, \tag{29}$$

where $\tilde{\omega}_1 = \tilde{z}_1(r_{d1}q_1;\chi)$, and q_1, $\bar{z}_1$, σ_1, $\tilde{z}_1$, π_2 are given by 7-13. Differentiating 29 w.r.t. χ gives the first order condition for profit maximum:

$$\begin{aligned} \frac{d\Pi}{d\chi} &\equiv -\frac{m}{n}\frac{1}{X}\left[\begin{array}{c} (\mu - r_{d1})(1-\Phi(\tilde{\omega}_1)) + \frac{\sigma_\chi^2}{2\sigma_1}\varphi(\tilde{\omega}_1) \\ +p_l(1-\delta_l)[(1-\delta_l)R_2(\chi,\delta_l) - y(1+a)](1-\Phi(\tilde{\omega}_1)) \\ +(\mu - r_{d1} + \frac{\sigma_\chi^2}{2\sigma_1}\tilde{\omega}_1)\frac{1}{\sigma_1}\varphi(\tilde{\omega}_1)\pi_2 \end{array}\right] \\ &= 0, \end{aligned} \tag{30}$$

where $\mu = (1-\bar{\delta})\chi$ is the expected return of the marginal project, and $\sigma_\chi^2 = (1-\bar{\delta})\chi(\chi - \frac{2n}{m}\bar{z}_1)$ is the contribution of the marginal loan to the total variance. Adding the marginal projects does not always increase the portfolio variance, because a reduction in χ reduces the part of the variance that arises because the projects are drawn from a distribution. However, as the bank will take too little risk if $\sigma_\chi^2 < 0$, I will focus on the case in which $\bar{\delta}$ is high enough for σ_χ^2 to be positive. Since $R_2(\chi,\delta_l) \geq y(1+a)$ and $\tilde{\omega}_1 < 0$, it follows directly that

$$\xi \equiv \mu - r_{d1} + \frac{\sigma_\chi^2}{2\sigma_1}\tilde{\omega}_1 < 0, \tag{31}$$

in profit maximum. Hence, marginal projects contribute positively to the probability of bank failure.

Consider how the deposit rate required to give depositors an expected return equal to y, denoted by $\tilde{r}_{d1}(\chi)$, changes with the marginal project. From the analysis in section A.1 follows that the IR condition of investors can be written as:

$$\rho(r_{d1},\chi) \equiv r_{d1}q_1(1-\Phi(\tilde{\omega}_1)) + \bar{z}_1\Phi(\tilde{\omega}_1) - \sigma_1\varphi(\tilde{\omega}_1) - y(1+a)q_1 = 0.$$

Since $\frac{\partial\rho}{\partial r_{d1}}$ is positive, the sign of $\frac{d\tilde{r}_{d1}}{d\chi}$ is the opposite of the sign of $\frac{\partial\rho}{\partial\chi}$, according to the Implicit Function Theorem. We have that

$$\frac{\partial\rho}{\partial\chi} = -\frac{m}{n}\frac{1}{X}[r_{d1}(1-\Phi(\tilde{\omega}_1)) + \mu\,\Phi(\tilde{\omega}_1)) - \frac{\sigma_\chi^2}{2\sigma_1}\varphi(\tilde{\omega}_1) - y(1+a)]. \tag{32}$$

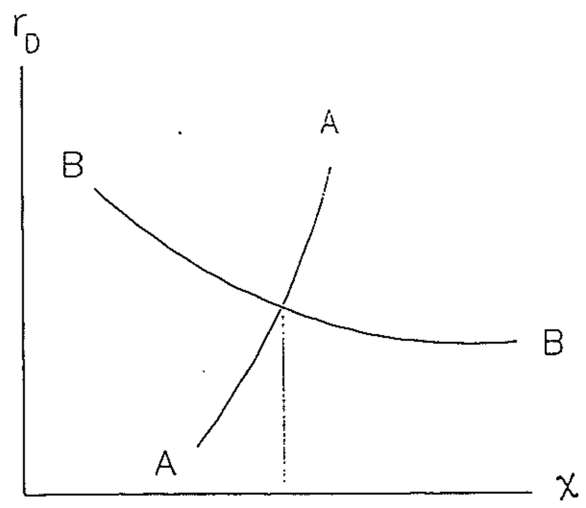
Substituting the IR condition into 32 gives

$$\frac{\partial\rho}{\partial\chi} = \frac{m}{n}\frac{1}{X}\left[(\frac{\bar{z}_1}{q_1} - \mu\,)\Phi(\tilde{\omega}_1)) + (\frac{\sigma_\chi^2}{2} - \frac{\sigma_1^2}{q_1})\frac{1}{\sigma_1}\varphi(\tilde{\omega}_1)\right].$$

Since $R_1(\tilde{\delta}, x) > \chi$ for all $x > \chi$, the first term is positive. The second term is negative, since in this model adding the marginal project decreases the average variance. It depends on the parameters of the model which effect dominates. I focus on the case when $\frac{\partial \rho}{\partial \chi} > 0$. In this case the locus of points in the $(\chi,\ r_{d1})$ plane which fulfils $\rho(r_{d1}, \chi) = 0$ is a downward-sloping curve (see the BB-curve in figure 1), with $\tilde{r}_{d1}(\chi) \geq y(1+a)$ for all χ.

As follows from 30, for a sufficiently profitable and diversified bank, $\frac{\partial^2 \Pi}{\partial \chi \partial r_{d1}}$ is dominated by the term $\frac{m}{n}\frac{1}{\chi}(1 - \Phi(\tilde{\omega}_1))$, which is positive. Moreover, the second order condition for profit maximum is fulfilled if the bank is sufficiently profitable and diversified. Hence, for a sufficiently profitable and diversified bank, the locus of points in the $(\chi,\ r_{d1})$ plane which fulfils equation 30 is an upward-sloping curve (see the AA-curve in figure 1). Since the deposit rate which fulfils 30 for low values of χ is smaller than $y(1 + a)$, it is clear that it exists an equilibrium for a sufficiently profitable and diversified bank.

Figure 1



A.4 Proof of lemma 1

Since $\rho(r_{d1}, \chi)$ does not depend on π_2, the equilibrium value of χ increases with π_2 as long as $\frac{d\Pi}{d\chi}$ does so (the AA- curve in figure 1 shifts outwards). From 30 follows that $\frac{d\Pi}{d\chi}$ increases with π_2 as long as the marginal project increases the probability that the bank will go bankrupt, that is as long as 31 is fulfilled. Since $\xi < 0$ for $\mu = y(1 + a)$ there is size of expected future profits for which the bank chooses $\mu > y(1 + a)$. Q.E.D.

A.5 Proof of proposition 3

I will show that there is a size of the expected second period profits for which the bank takes less risk in the sense of lemma 1 when $\theta = \theta_b$ than when $\theta = 1$. I will consider the case when θ only affects the value of χ. The motivation is that if the proposition holds in this situation, it will hold for the case when θ affects the probability of success. The reason is that the variance of the portfolio return increases with θ when θ affects χ, while the variance decreases with θ, when θ affects the probability of success.

A perfectly safe bank will compensate for the decrease in θ by increasing χ, but not fully as the return will be normal in the second period. What happens with the risk-incentive of the bank captured by the two last terms in 14 when $\theta\chi$ decreases? The derivative of 32 w.r.t. χ is given by

$$-\frac{m}{n}\frac{1}{X}\left[\xi\varphi(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{d\chi} + (1-\bar{\delta})\Phi(\tilde{\omega}_1) - \frac{d}{d\chi}(\frac{\sigma_\chi^2}{2\sigma_1})\varphi(\tilde{\omega}_1)\right], \tag{33}$$

which is negative, since I focus on the case when the effect on the variance is small. In that case, the bank has stronger incentive to exploit its limited liability in bad times. However, the contribution of the marginal project to the probability of bank failure decreases with χ:

$$\frac{m}{n}\frac{1}{X}\left[\xi\frac{d}{d\chi}(\frac{1}{\sigma_1}\varphi(\tilde{\omega}_1))\pi_2 + \left[(1-\bar{\delta}) + \frac{d}{d\chi}(\frac{\sigma_\chi^2}{2\sigma_1}\tilde{\omega}_1)\right]\frac{1}{\sigma_1}\varphi(\tilde{\omega}_1)\pi_2\right] > 0, \tag{34}$$

The sum of 33 and 34 is positive when

$$\begin{aligned} & \left(\frac{1}{\sigma_1}(-\tilde{\omega}_1)\pi_2 - 1\right)\xi\varphi(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{d\chi} + \xi\frac{d}{d\chi}(\frac{1}{\sigma_1})\varphi(\tilde{\omega}_1)\pi_2 \\ + \quad & (1-\bar{\delta})\left(\frac{1}{\sigma_1}\varphi(\tilde{\omega}_1)\pi_2 - \Phi(\tilde{\omega}_1)\right) + \frac{d}{d\chi}(\frac{\sigma_\chi^2}{2\sigma_1})\varphi(\tilde{\omega}_1)\left(\tilde{\omega}_1\frac{1}{\sigma_1}\pi_2 + 1\right) \\ + \quad & \frac{\sigma_\chi^2}{2\sigma_1}\frac{d\tilde{\omega}_1}{d\chi}\frac{1}{\sigma_1}\varphi(\tilde{\omega}_1)\pi_2 > 0. \end{aligned} \tag{35}$$

Since, according to lemma 1, $\frac{1}{\sigma_1}\varphi(\tilde{\omega}_1)\pi_2$ and χ increase with π_2, and $\tilde{\omega}_1$ and r_{d1} decrease with π_2, there is a size of π_2 for which 35 is positive, given that the effect on the variance is small. Moreover, the deposit rate will be higher in bad times, which increases the incentive to exploit limited liability, $\frac{\partial^2 \rho}{\partial\chi\partial r_{d1}} > 0$, but also the expected bankruptcy costs, and again there is a size of π_2 for which the latter effect will dominate. Hence, we can conclude that there is a size of future expected profits for which the bank will have less incentive to take risk in bad times in the sense that the sum of the two last terms in 14 will be larger in equilibrium when $\theta = \theta_b$ than when $\theta = 1$. *Q.E.D.*

A.6 Proof of proposition 6

I will show that there is a size of the expected second period profit for which an increase in n leads to higher equilibrium value of χ, if the bank is sufficiently large and profitable. Since $\rho(r_{d1}, \chi)$ decreases with n (the BB-curve shifts upwards, see figure 2), the equilibrium value of χ will increase with n, if $\frac{d\Pi}{d\chi}$ increases with n (the AA-curve shifts outwards, see figure 2). Thus, I will show that there is a size of π_2 for which $\frac{\partial^2 \Pi}{\partial\chi\partial n} \geq 0$ in equilibrium.

Note that it is sufficient to evaluate the derivative of the term within the square brackets in 30, since this term is zero in equilibrium. Let us start with the effect on the expected bankruptcy cost. Note that 31 is independent of n. Thus, the change in the expected bankruptcy cost is determined by

$$\frac{d}{dn}(\frac{1}{\sigma_1}\varphi(\tilde{\omega}_1)\pi_2) = -\frac{1}{2n\sigma_1}\varphi(\tilde{\omega}_1)\pi_2(1-\tilde{\omega}_1^2),$$

which is larger than zero, if $\tilde{\omega}_1 < -1$. Furthermore, if $(1-\delta_l)R_2(\chi, \delta_l) > y(1+a)$, a larger n implies a smaller effect on the customer stock. Hence, for $\tilde{\omega}_1 < -1$ the contribution of the marginal loans to expected future profits decreases as n increases, which gives the bank incentive to raise χ.

On the other hand, the contribution of the marginal loans to current profits increases as n increases: To see this differentiate the first two terms of 30 with respect to n :

$$-(\mu - r_{d1})\varphi(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{dn} + \sigma_\chi^2\left(\frac{1}{2\sigma_1}\varphi'(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{dn} - \frac{1}{4\sigma_1^2\sigma_1}\frac{d\sigma_1^2}{dn}\right)\varphi(\tilde{\omega}_1),$$

which can be rewritten as

$$-\xi\varphi(\tilde{\omega}_1)\frac{d\tilde{\omega}_1}{dn} + \sigma_\chi^2\frac{1}{4\sigma_1^2}\frac{1}{n}\varphi(\tilde{\omega}_1),$$

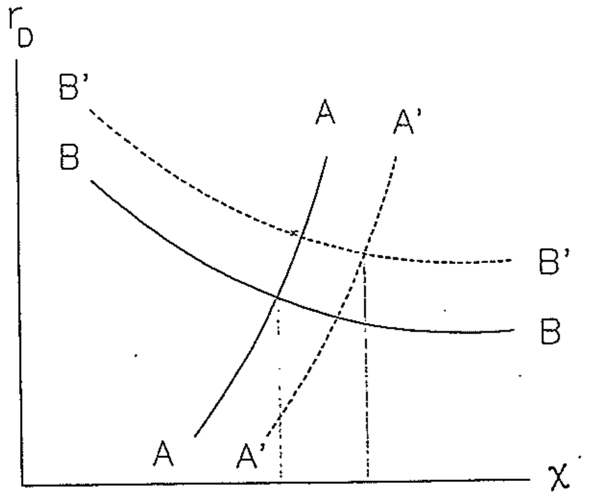
which is positive in profit maximum. Hence, $\frac{\partial^2 \pi_1}{\partial\chi\partial n} < 0$.

The effect on future profits dominates the effect on current profits when

$$\xi\frac{1}{\sigma_1}\varphi(\tilde{\omega}_1)\pi_2(1-\tilde{\omega}_1^2) - p_l(1-\delta_l)\left((1-\delta_l)R_2(\chi,\delta_l) - y(1+a)\right)\tilde{\omega}_1\varphi(\tilde{\omega}_1) \geq$$
$$\xi\ \ \tilde{\omega}_1\varphi(\tilde{\omega}_1) + \sigma_\chi^2\frac{1}{2\sigma_1}\varphi(\tilde{\omega}_1).$$

From the proof of lemma 1, we know that for $\tilde{\omega}_1 < -1$, the first term increases with π_2, while all the other terms decrease with π_2 as both $\varphi(\tilde{\omega}_1)$ and $\tilde{\omega}_1\varphi(\tilde{\omega}_1)$ approach zero as $\tilde{\omega}_1$ decreases. Hence, for a sufficiently large and profitable bank, there is a size of π_2 for which the first effect dominates, and $\frac{\partial^2\Pi}{\partial\chi\partial n} \geq 0$ in equilibrium. *Q.E.D.*

Figure 2



B Appendix

In this appendix, I consider the case when established entrepreneurs have invested some of their first period return in durable goods, which can be pledged as collateral in the second period. A loan contract with a collateral requirement states that either the borrower pays the lender the stated loan rate and keeps his collateral, or the borrower defaults and the collateral accrues to the lender. I assume that the collateral is more highly valued by the owner than by other agents. This means that collateral requirements are costly. Denote a given collateral requirement by C, and let βC be the value of the collateral to the lender, where $\beta \in (0, 1)$. I assume that the cost of providing collateral is sufficiently low, for low-risk entrepreneurs to prefer to provide collateral in order to separate them from high-risks entrepreneurs than to be pooled together with high-risks.

I allow the bank to give a take it or leave it offer to the entrepreneur, and I assume that whenever the entrepreneur is indifferent with respect to loan offers, his first choice is to accept the offer of the bank which financed his project in the previous period. As for the rest, the analysis is based on the same assumptions as the analysis in the main text.

For contracts to be separating, high-risks must prefer one of the contracts, (r_h, C_h), and low-risks must prefer the other contract, (r_l, C_l), where C is the collateral requirement:

$$(1-\delta_h)r_h + \delta_h C_h \leq (1-\delta_h)r_l + \delta_h C_l, \quad (36)$$
$$(1-\delta_l)r_l + \delta_l C_l \leq (1-\delta_l)r_h + \delta_l C_h. \quad (37)$$

In equilibrium each investor offers the set of contracts that minimizes the sorting cost, $(1-\beta)(\delta_h C_h + \delta_l C_l)$, subject to 36, 37 and

$$\begin{aligned} (1-\delta_h)r_h + \delta_h \beta C_h &= y, && (38) \\ (1-\delta_l)r_l + \delta_l \beta C_l &= y. && (39) \end{aligned}$$

If the contract earned more than y, or involved a larger sorting cost than the minimum, there would be a deviating contract that attracted entrepreneurs and gave the lender a higher return than y. Furthermore, there are sufficiently many high-risks, and the sorting cost is sufficiently low, for there not to be any pooling contract which gives an expected return of y to the investor, and which low-risks prefer to the separating contracts.

The problem of minimizing $(1-\beta)(\delta_h C_h + \delta_l C_l)$ subject to 36-39, and the feasibility constraints on r_h, r_l, C_h and C_l, has the unique solution[36]

$$\begin{array}{ll} \hat{r}_h = \frac{y}{1-\delta_h}; & \hat{C}_h = 0, \\ \hat{r}_l = \frac{y-\delta_l\beta\hat{C}_l}{1-\delta_l}; & \hat{C}_l = y\frac{\delta_h-\delta_l}{\delta_h-\delta_l+(1-\beta)(1-\delta_h)\delta_l}. \end{array} \qquad (40)$$

The intuition is as follows. Assume that types are known, and that each type pays a loan rate that gives an expected return of y to the investor. Since the default probability of a high-risk is higher than the default probability of a low-risk, the loan rate of high-risks is higher than the loan rate of low-risks. Thus, offering these contracts when types are not known to the lender, leads to both types choosing the contract of low-risks, and the expected return to the lender being less than y. This means that the contracts should be changed so that high-risks prefer the high-risk contract, and low-risks continue to prefer the low-risk contract. Since a given collateral requirement is more onerous to high-risks than to low-risk, the high-risk contract should not involve a collateral requirement. The low-risk contract should involve just enough collateral for high-risks to prefer the high-risk contract. Then, low-risks still prefer the low-risk contract.

Investors offer the contracts given by 40 to established entrepreneurs, independently of the offers of banks. Because the contracts are separating, the return to the investor does not depend on the number of low-risks and high-risks that ask for direct loans.

Since collateral requirements are costly, the bank can offer a loan contract to its low-risk customers, which earns a higher return than y, but which still makes the

[36]The proof is similar to the proof of Proposition 2 in Besanko and Thakor (1987), and is therefore omitted. It requires that the wealth of every established low-risk entrepreneur for which $x \geq \hat{r}_l$ is at least $\hat{C}_l$. For simplicity, I assume that at least $\hat{C}_l$ of the input requirement of the project is a remuneration to the entrepreneur for his work on the project.

entrepreneur at least as well off as if he funds his project in the capital market.[37] Denote the loan rate offered by banks to their low-risk customers by r_l^b. Let $\hat{r}_l^b$ be the value of r_l^b which is such that

$$(1-\delta_l)\hat{r}_l^b = (1-\delta_l)\hat{r}_l + \delta_l \hat{C}_l. \tag{41}$$

Thus, the expected cost to a low-risk entrepreneur of a loan contract stating the loan rate $\hat{r}_l^b$ is equal to his expected cost of financing his project in the capital market. If x is less than $\hat{r}_l^b$ for some established entrepreneurs, the expected return on loans to these entrepreneurs is $(1-\delta_l)x$. Thus, we have that $R_2(x, \delta_l) = \hat{r}_l^b$, for $x \geq \hat{r}_l^b$, and $R_2(x, \delta_l) = x$, for $x < \hat{r}_l^b$.

Substituting the expression for $\hat{r}_l$ given by 40 into 41 gives

$$(1-\delta_l)\hat{r}_l^b = y + \delta_l(1-\beta)\hat{C}_l.$$

This expression implies that banks earn informational rents on their low-risk customers. Given that the expected informational rent, which is equal to the expected sorting cost, $\delta_l(1-\beta)\hat{C}_l$, is larger than the cost of financial intermediation per invested unit, ya, it is easy to show that the results hold for this specification of the model as well. However, there is a higher social cost of bank failure in this model specification. If the bank goes bankrupt in the first period, its low-risk customers have to borrow directly from investors in the second period, and each low-risk entrepreneur has to pay a sorting cost in the capital market. This affects the welfare results, but, as I will show, there are still two counteracting forces on the risk-taking behaviour of banks.

The expected social surplus created by the bank is given by

$$\begin{aligned} W(\chi) &= S_1(\chi) - y(1+a)q_1(\chi) + [S_{l2}(\chi) - y(1+a)q_{l2}(\chi)](1-\Phi(\tilde{\omega}_1)) \\ &\quad \left[S_{l2}(\max(\hat{r}_l^b, \chi)) - (y + \delta_l(1-\beta)\hat{C}_l)q_{l2}(\max(\hat{r}_l^b, \chi))\right]\Phi(\tilde{z}_1(\tilde{\omega}_1)) \\ &\quad S_{h2}(\max(\hat{r}_h, \chi)) - yq_{h2}(\max(\hat{r}_h, \chi)), \end{aligned} \tag{42}$$

where $\tilde{\omega}_1$, S_1, q_{l2}, q_{h2}, S_{l2}, S_{h2} are defined by 16-19 and q_1 by 7. The terms in 42 can be explained as follows. In the first period all projects are financed with bank loans at the social funding cost $y(1+a)$. In the second period low-risk projects, if possible, are financed with bank loans. If the bank goes bankrupt, those of its low-risk customers who find it worthwhile borrow directly from investors in the second period, which gives rise to sorting costs. High-risk projects with sufficiently large x are financed with direct loans from investors at the social funding cost y, in the second period.

[37]There is some empirical support for the idea that the bank can earn rents by not asking for collateral. Berger and Udell (1995) find that borrowers with longer banking relationships pay lower interest rates and are less likely to pledge collateral.

Let us first consider the case where $\hat{\chi} > \hat{r}_h$. Differentiating W with respect to χ, and then substituting the equilibrium condition into the first order condition for a socially optimal lending policy gives

$$\begin{aligned} \frac{dW}{d\chi}_{\hat{\chi}>\hat{r}_h} &= -\frac{m}{n}\frac{1}{X}[(p_l(1-\delta_l)(1-\delta_l)(\hat{\chi}-\hat{r}_l^b)+p_h(1-\delta_h)((1-\delta_h)\hat{\chi}-y)] \\ &\quad +\frac{\partial\rho}{\partial\chi}-\pi_2\varphi(\tilde{\omega}_1)\frac{d\tilde{z}_1}{dr_{d1}}\frac{dr_{d1}}{d\chi}. \end{aligned} \quad (43)$$

The first two terms in 43 represent the part of the surplus of the marginal projects that the bank does not take into account when determining its optimal lending policy. Competition from the capital market restricts the surplus of low-risks that the bank can appropriate, and the bank does not consider the expected surplus of high-risk marginal projects in the second period, since these projects will be funded in the capital market. The last two terms in 43 represent the effect of the change in the deposit rate, which the bank does not take into account. Thus, there are counteracting effects on welfare, and the outcome is not obvious.

Let us now consider the case when $\hat{\chi} < \hat{r}_l^b$. Then the value of the derivative of W with respect to χ in equilibrium is given by

$$+\frac{\partial\rho}{\partial\chi}-\pi_2\varphi(\tilde{\omega}_1)\frac{d\tilde{z}_1}{dr_{d1}}\frac{dr_{d1}}{d\chi},$$

which is positive. In this case the bank takes too much risk. The social cost of bank failure is equal to the private cost of bank failure. Moreover, there is no undervaluation due to lack of commitment, because the bank can appropriate the whole second period surplus as the entrepreneur has no alternative funding. Hence, remains only the moral hazard problem.

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